A View of Mathematics

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The purpose of these lectures is to equip students with ideas to take to the primary school classroom, and enable them to convey the beauty, importance and relevance of the subject with enthusiasm.

I emphasize five main words in my last sentence – ideas, beauty, importance, relevance and enthusiasm. Needless to say, these five aspects go hand in hand, and it is difficult if not impossible to imagine having one without any of the others.

Of these five words, perhaps the most important is the word enthusiasm. Any subject, no matter how exciting or interesting, would come across as boring if it were taught by someone who could not raise any enthusiasm about the subject matter. You have the very important and noble task of getting young children interested in mathematics. We all know that they will do well and want to do more if they are excited and enjoy doing it. It follows that if we can convey our enthusiasm of the subject to them, then we are already well on the way towards a successful outcome. But if I want you to be enthusiastic about the subject, then it is up to me and my colleagues to convince you that it is a wonderful subject, full of beautiful ideas, and of great importance and relevance to many aspects of everyday life. To put it succinctly, if I and my colleagues cannot get you excited about the subject, then we cannot expect you to be able to get the children excited about mathematics.

It is all very well to say that we want to be enthusiastic about mathematics. However, for one to be enthusiastic about the subject, one must also have a good understanding of some of its basic features as well as the material that one is going to teach. To have a good understanding, one must learn a great deal more than what one is going to teach. Teaching at the limit of one's knowledge is always going to be extremely difficult, and getting the material across correctly is always going to be at the expense of other equally important aspects of the teaching process.

The purpose of these lectures is not to equip you with the material that you are going to teach, but rather to equip you with ideas and insight into how best to get that material across in an exciting and enthusiastic way. We have the luxury of choosing topics that will allow us to bring out some of the best features of the subject, and we are going to choose material that will fit some or all of the following important criteria, listed in no particular order:

- It illustrates some of the beautiful aspects of the subject.
- It helps us understand or appreciate interesting phenomena in the real world.
- It helps us to ask interesting questions.
- It enables us to appreciate the role of mathematics in our everyday life.
- It provides a tool to enable us to analyze concrete problems in a systematic way, to understand the results, and to appreciate the implications of our assumptions.
- It illustrates how mathematics can be made relevant to the problems we try to understand.
- It highlights the need to develop our theories in an appropriate way.
- It highlights the importance of proper analysis to fully appreciate results that may be counter intuitive.
- It is full of beautiful ideas.

The topics that we shall discuss will include some or all of the following, listed in no particular order:

- Arithmetic and numbers – We try to understand some of the basic features of arithmetic, as well as some of the number systems that have historical significance, and others that underpin modern technology such as computing and information security.

- Relationship between numbers and geometry – We discuss how the golden ratio arises from the study of a number sequence and how it features in important aspects of geometry.

- Basic geometry – We try to understand how far we can go with ruler and compass subject to a set of very basic rules.

- Geometric insight – Much of mathematics is motivated by considerations of geometry. We study how some simple geometric intuition can lead us to important ideas in mathematics.

- Geometry, symmetry, polygons and solids – We appreciate the idea of symmetry which is present in many of the geometric shapes that surround us.

- Fractals – We see some of the modern aspects of geometry that give us extra insight into the real world.

- Mathematics of perspective drawing – We appreciate how mathematics is used in architecture as well as the arts.

- Topology – We appreciate how simple ideas on untangling can make a seemingly complicated problem rather straightforward.

- Probability – This is a topic that has to be properly done, otherwise we end up making awful mistakes.

- Graphs and networks – We want to understand how networks can be analyzed properly by studying graph theory in a systematic way.

- Geodesics or paths – We appreciate how some problems in 3-dimensional space can best be studied by reducing the problem to one in 2-dimensional space where our sense of geometry is much stronger and better developed.

- Logic and paradoxes – We appreciate the basic structures of a logical argument.

Some of the topics will be discussed in a reasonably systematic way, while others will come across as simple puzzles that help to illustrate either a very good idea or an important point in an amusing way.