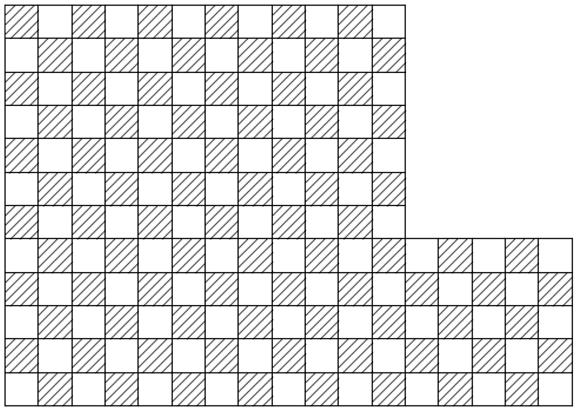
The Problem of Mrs Pythagoras – courtesy of Sam Loyd

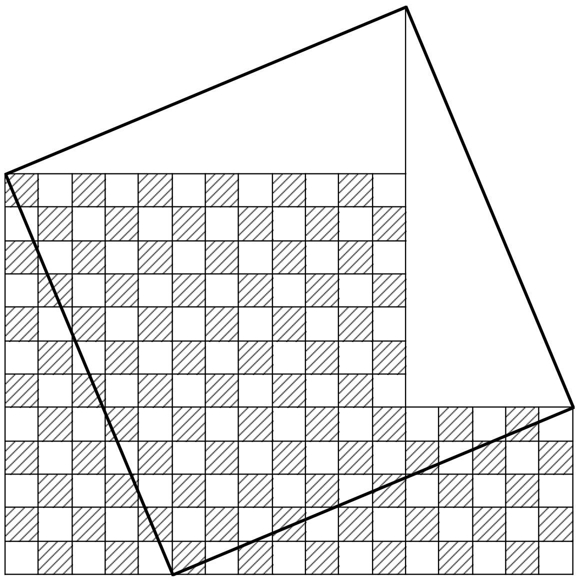
William Chen

A matting is made up of two squares, of dimensions 12×12 and 5×5, joined together as shown.



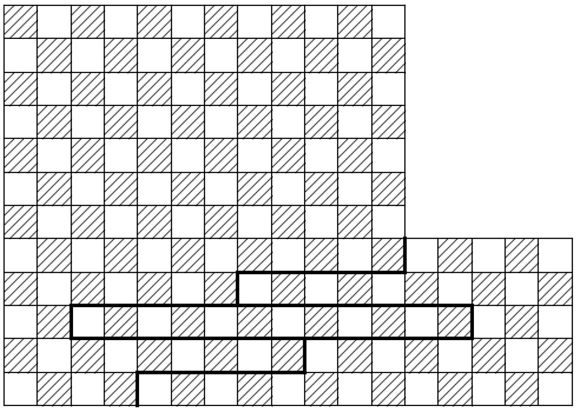
The question is to convert this into a square matting by cutting it into three pieces and reassembling. It turns out that there are a number of ways to do this, some better than others.

First of all, we note that there are $12^2 + 5^2 = 169 = 13^2$ little squares, so we should end up with a square with dimension 13×13. Pythagoras may proceed along the following lines. He realizes that 13 is the length of the hypothenuse of a right triangle with shorter sides 12 and 5, so he will try to draw such a triangle in the picture. The following is then a possible solution.

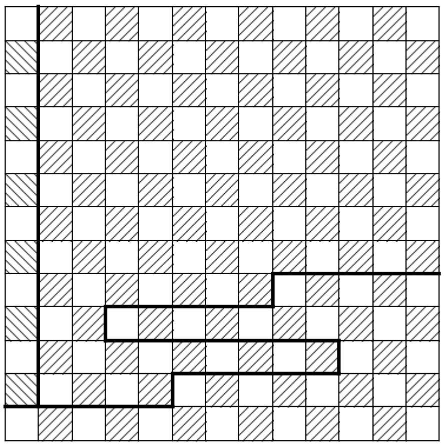


Now he moves the triangle in the bottom right to the top left, and the triangle in the bottom left to the top right. But then Mrs Pythagoras may object, since this process involves cutting up some of the little squares, and the resulting matting is not very pleasing to look at.

Not to worry. Perhaps we can throw the theorem of Pythagoras away and try the following instead. We cut the matting into three pieces as shown.

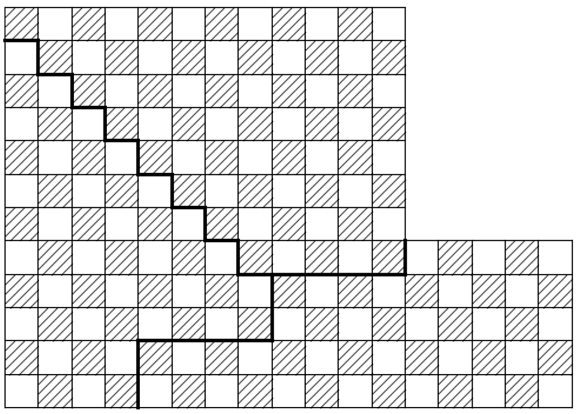


We then move the narrow strip to the left, with a white square on top. Then we place the other piece below and it all fits nicely.



Mrs Pythagoras still is not satisfied. Note that most of the dark squares have threads from NE to SW. But the piece on the left, having been rotated 90 degrees, now have dark squares with threads from NW to SE.

Now observe that the square of dimension 12×12 have equal numbers of white and dark squares, whereas the square of dimension 5×5 has one more white square than dark squares. So a perfect squares of dimension 13×13 must have one more white square than dark square, and so the top left square must be white. If we look at the square of dimension 12×12, it appears that our only hope is to somehow move the white square below the top left dark square to the left of this top left dark square. This is the motivation of the following method.



We now reassemble the three pieces and arrive at the following perfect solution.