Honey, Fly and Spider – courtesy of Henry Ernest Dudeney

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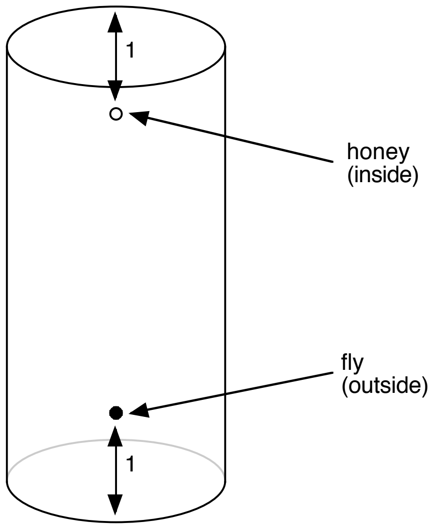
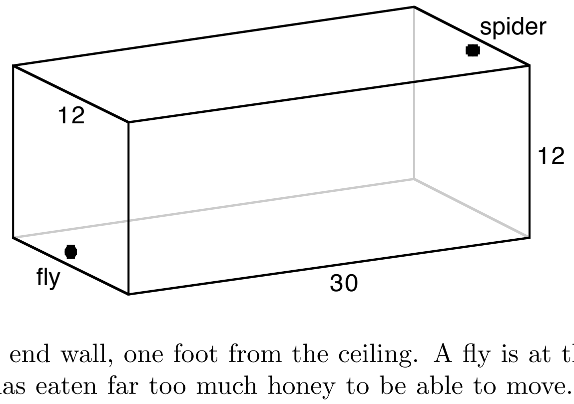
Henry Ernest Dudeney is arguably the greatest inventor of puzzles who has ever lived. Here we discuss the simple mathematics behind two of his geodesic problems.

The first and simpler of our two problems concerns a cylindrical glass, 4 inches high and 6 inches in circumference, as shown in the picture below.



On the inside, one inch from the top, is a drop of honey. On the outside, one inch from the bottom and directly opposite, is a fly. What is the shortest path by which the fly can crawl to the honey, and exactly how far does the fly crawl?

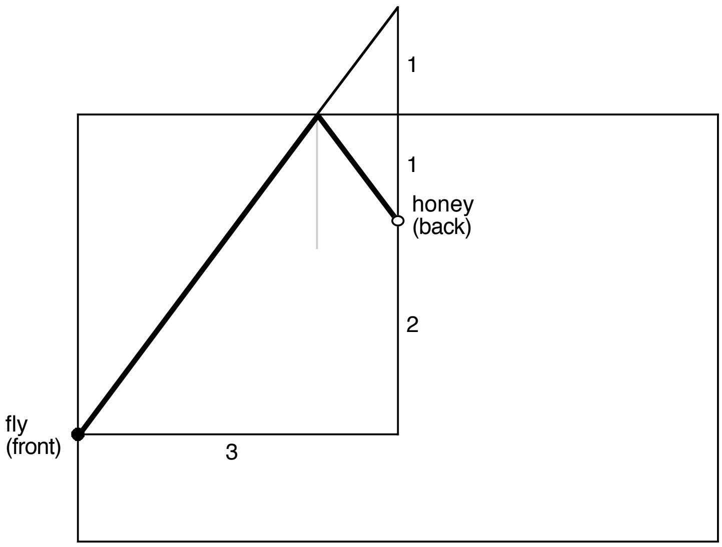
The second and more difficult of our two problems concerns a rectangular room, 30 feet long, 12 feet wide and 12 feet high, as shown in the picture below.



A spider is at the middle of an end wall, one foot from the ceiling. A fly is at the middle of the opposite wall, one foot above the floor, and has eaten far too much honey to be able to move. What is the shortest distance the spider must crawl in order to reach the fly?

The solution to each of these problems involves the simple technique of cutting part of the surface in a suitable way and flattening the cut surface. In this way, the problem becomes a problem on the plane.

For the first problem, we do not have to worry about the bottom of the cylindrical glass, as clearly the fly is going to crawl up the outside of the glass, over the rim and then inside. The idea is to give the vertical side of the glass a cut, along a vertical line that passes through the point where the fly is, and then roll out the side to obtain the situation shown below.



Then clearly the shortest path is the most direct path as shown – the fly crawls up the outside, over the rim and crawls down the inside in a diagonal path, and the two parts of the diagonal path make the same angle with the vertical. Clearly the vertical distance travelled by the fly is 4 feet – 3 feet up and one foot down, and the horizontal distance travelled by the fly is 3 feet, half the circumference. The length of the path is the length of the hypothenuse of a right angled triangle with shorter sides of 3 and 4 feet, and so must be 5 feet if we use Pythagoras's theorem.

For the second problem, we pretend that the floor, walls and ceiling of the room form a rectangular cardboard box. We then cut the cardboard box along some of its edges in order to flatten it, and then draw a straight line segment to join the spider and the fly. However, there are a few different ways by which we can achieve this. The shortest path is then the one where the straight line segment joining the spider and the fly is smallest. This best solution is summarized by the picture below.