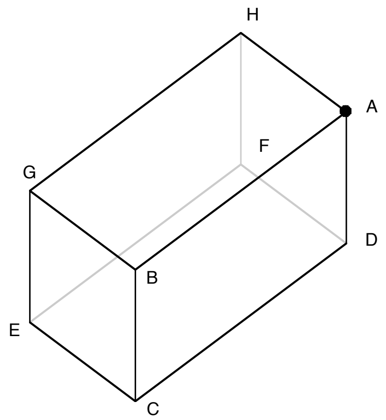
How to be Kind and Cruel to a Hungry Ant – courtesy of Kevin Buzzard

William Chen

A hungry ant is sitting at a corner of a big rectangular box, 2 metres by 1 metre by 1 metre. In the picture below, the corner A is where the ant is.



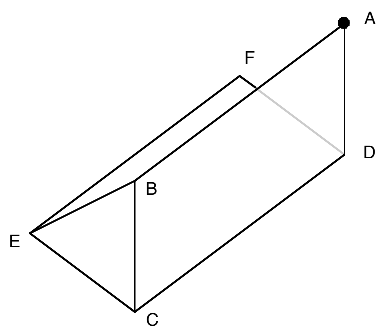
We now want to be kind to the ant, and stick some food somewhere on the surface of the box. However, we also want to be cruel to the ant, so we want to make it travel the furthest to get to the food. Where on the surface of the box do we stick the food?

Bear in mind that the ant can only travel on the surface of the rectangular box. It cannot go through the middle.

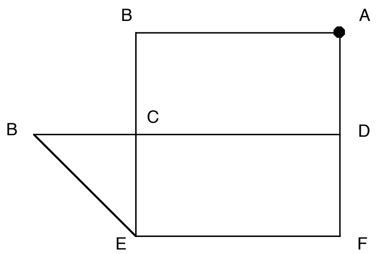
First of all, it is quite obvious that we do not stick the food anywhere on the face ADFH. On this face, the furthest point from the point A is the corner F, but the point C is clearly further from the point A than the point F is.

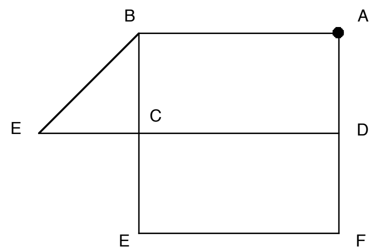
Next, we do not need to consider the face ABGH, as long as we consider the face ABCD. To see this, note that if we put a mirror to pass through the points ABEF, then the face ABGH is just the image of the face ABCD in the mirror. Similarly, we do not need to consider the face HGEF, as long as we consider the face DCEF, as the face HGEF is the image of the face DCEF in the same mirror. Furthermore, on the face BCEG, we need not consider the triangular half-face BEG, as long as we consider the triangular half-face BEC, as the half-face BEG is the image of the half-face BEC in the same mirror.

We therefore remove the faces ADFH, ABGH, HGEF and the triangular half-face BEG. In the picture below, we have the remaining parts of the original rectangular box.

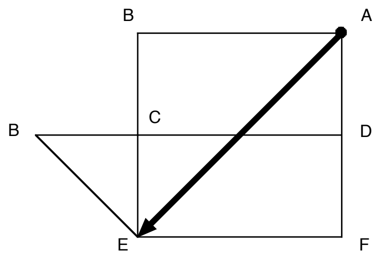


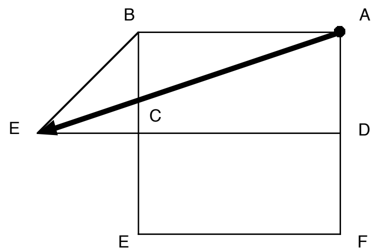
Our first important idea here is to flatten what remains of the rectangular box. We may cut it along the edge BC to obtain the picture below on the left, or cut it along the edge EC to obtain the picture below on the right.



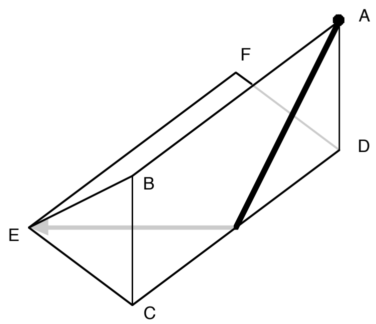


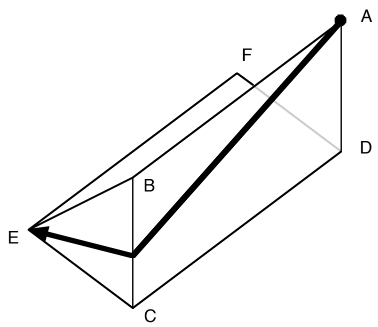
These two pictures show immediately that we do not stick the food on the face ABCD or the face DCEF, except possibly at the corner E. On these two faces, the furthest point from the point A is the corner E. The following pictures show how this corner E can be reached.





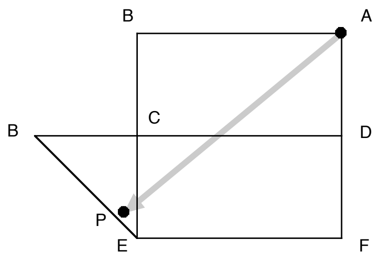
These correspond to the pictures below. The ant is well advised to take the strategy on the left!

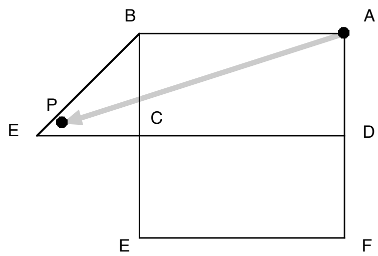


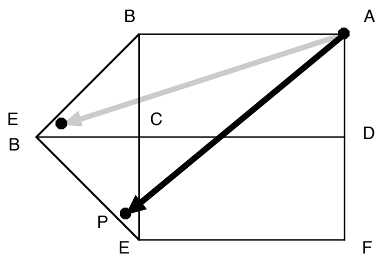


Since the corner E is on the triangular face BCE, we therefore conclude that we must stick the food somewhere on this face. We shall show that we must place the food somewhere on the line segment BE. Our second important idea here is to show that any point on the face BCE that is not on the edge BE **cannot possibly be** furthest from the point A. Take any point P on this face which is not on the edge BE. We have one of the following four possibilities. For each of these four possibilities, we draw three pictures. The first picture illustrates the case when we cut along the edge BC, the second picture illustrates the case when we cut along the edge EC, and the third picture compares these two cases, and highlights the shorter route from the point A to the point P.

(1) The first possibility is illustrated by the three pictures below.

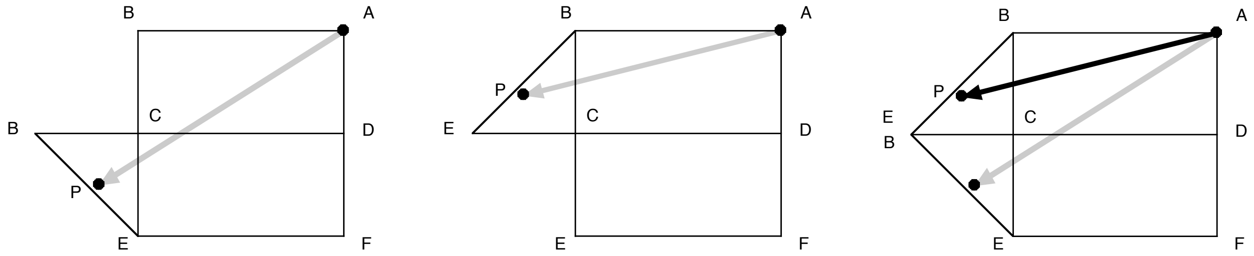






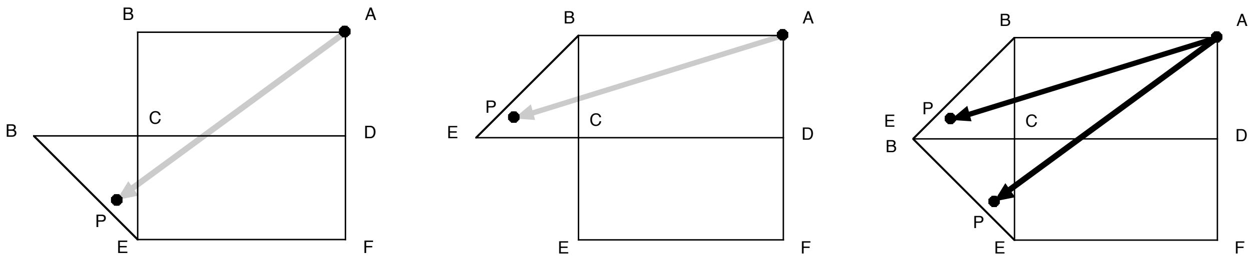
Note that the first picture gives the shorter route from the point A to the point P. If we now follow the dark arrow AP and go a little beyond P, we end up at a point which is a little further from the point A than the point P is. So the point P **cannot possibly be** furthest from the point A.

(2) The second possibility is illustrated by the three pictures below.



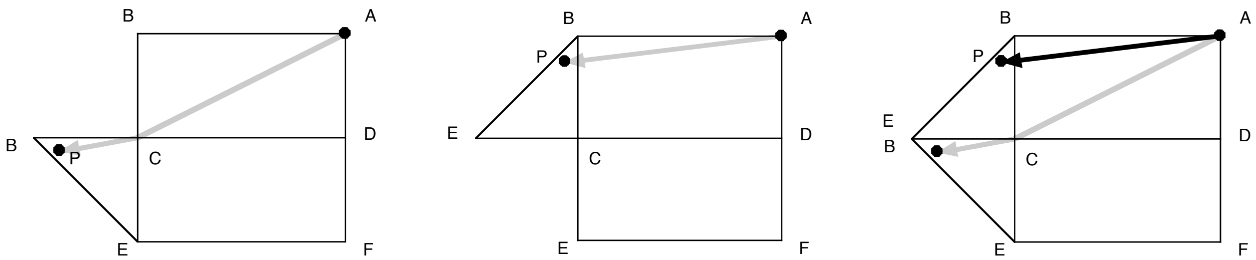
Note that the second picture gives the shorter route from the point A to the point P. If we now follow the dark arrow AP and go a little beyond P, we end up at a point which is a little further from the point A than the point P is. So the point P **cannot possibly be** furthest from the point A.

(3) The third possibility is illustrated by the three pictures below.



Note that the two different routes from the point A to the point P are equal in length. If we now follow either of the dark arrows AP and go a little beyond P, we end up at a point which is a little further from the point A than the point P is. So the point P **cannot possibly be** furthest from the point A.

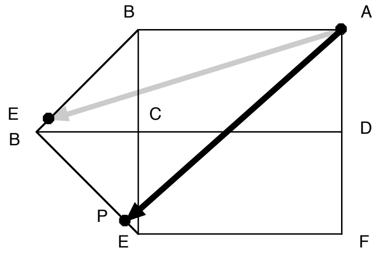
(4) The last possibility is illustrated by the three pictures below.

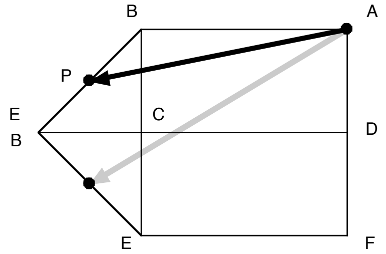


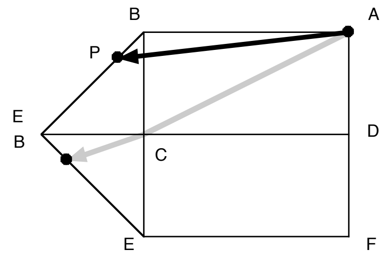
Note that the second picture clearly gives the shorter route from the point A to the point P. If we now follow the dark arrow AP and go a little beyond P, we end up at a point which is a little further from the point A than the point P is. So the point P **cannot possibly be** furthest from the point A.

Let us summarize what we have discussed so far. We have shown that the furthest point from A cannot be on the faces ADFH, ABCD or DCEF, except possibly the point E. By a mirror argument, the furthest point from A cannot be on the faces ABGH and HGEF, again except possibly the point E. We therefore conclude that the furthest point from A must be on the face BCEG. However, it cannot be in the triangle BEC, except possibly on the edge BE. By a mirror argument, it cannot be in the triangle BEG, again except possibly on the edge BE. So the furthest point from A **must be** on the line segment BE, as we have eliminated all the other points from contention!

We now investigate where on the line segment BE is the furthest point from A. Corresponding to possibilities (1), (2) and (4) above, we have the following pictures.

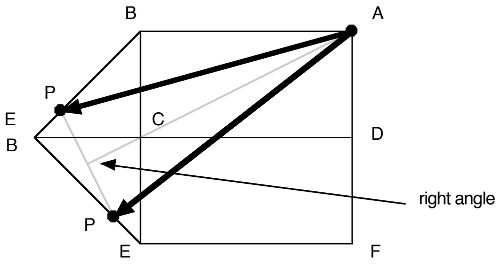




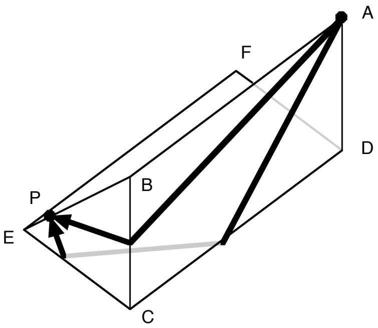


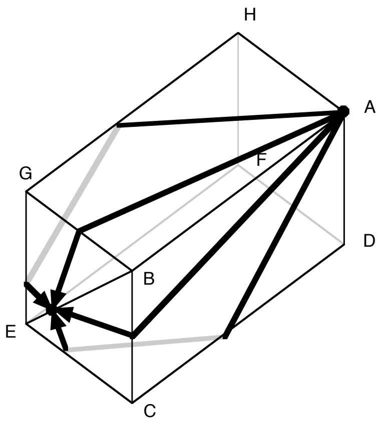
In the picture above on the left, the point P cannot be furthest from the point A. If we move a little from P along the edge towards the point B, we shall reach a point further away from the point A than the point P is. In the picture above in the middle, the point P cannot be furthest from the point A. If we move a little from P along the edge towards the point E, we shall reach a point further away from the point A than the point P is. In the picture above on the right, the point P cannot be furthest from the point A. If we move a little from P along the edge towards the point E, we shall reach a point further away from the point A than the point P is.

Only the possibility below remains. Here there are two best ways of reaching the point P.



We have drawn a line segment joining the positions of the point P corresponding to the two different cuts along the edge BC and along the edge EC. We have also taken the midpoint of this line segment, and drawn a line segment joining this to the point A. These two line segments are **perpendicular** to each other. Unflattening the picture above, we obtain the picture below on the left. We therefore stick the food at this point P. Going back to the original picture, we find that there are four ways the ant can reach the point P by travelling the shortest distance, as shown in the picture below on the right.





A little calculation will show that the point P is precisely at a quarter distance along the line segment EB from the corner E.

On the next page, you will find a drawing with which you can make up a rectangular box with the shortests paths for the ant shown.

Cut out the drawing below to make your rectangular box.